CALCULATING THE DISTRIBUTION OF A STREAM ALONG A CONTACT OR FILTERING APPARATUS

I. E. Idel'chik

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The formulas presented make it possible to calculate the flow distributions in Z-shaped contact, filtering, and similar apparatus with arbitrary specified geometric parameters.

We investigated a similar problem in [1]. However, formula (7) of [1] is valid for apparatus with a Z-shaped intake-discharge configuration (Fig. 1) only in the special case in which

$$\zeta_{cyl} = \zeta^*_{cyl} = 0 \text{ and } \frac{F_a}{F^*_a} = 1.$$

In the more general case in which the above quantities have specified values, the differential equation becomes

$$\bar{Q}'^2_x \pm A_0\bar{Q}^2_x \pm B_0\bar{Q}_x - \bar{Q}'^2_0 = 0, \tag{1}$$

where

$$A_0 = (1 \mp \zeta_{cyl})\mu^2\bar{f}^2\bar{S}^2 - (1 \pm \zeta^*_{cyl})\mu^2\bar{f}^2\bar{S}^{*2}; \tag{2}$$

$$B_0 = 2(1 \pm \zeta^*_{cyl})\mu^2\bar{f}^2\bar{S}^{*2}. \tag{3}$$

Here and below, the upper signs refer to injection and the lower signs to suction.

The first of these two parameters can have any sign, i.e.,

$$\mp A_0 > 0; \quad \mp A_0 < 0 \text{ and } A_0 = 0.$$

Each of these cases is associated with a different set of solutions which enable us to compute the distribution of relative velocities and pressures along the apparatus.

We write out the final computation formulas for the first two cases without derivation (the third case is presented in our first paper [1]).

Case I ($\mp A_0 > 0$):

$$\bar{Q}_x = \bar{W}_x = \frac{1}{B_1}\left[B_2 \exp\sqrt{\mp A_0}\,x - B_3\exp(-\sqrt{\mp A_0}\,x) - B_4\right], \tag{4}$$

$$\bar{v}_x = \frac{\sqrt{\mp A_0}}{B_0}\left[B_2\exp\sqrt{\mp A_0}\,x + B_3\exp(-\sqrt{\mp A_0}\,x)\right], \tag{5}$$

$$\Delta\bar{p}_x = \mp\frac{A_0}{B_1^2\mu^2\bar{f}^2\bar{S}^2}\left[B_2\exp\sqrt{\mp A_0}\,x + B_3\exp(-\sqrt{\mp A_0}\,x)\right], \tag{6}$$

$$\zeta_{tot} = \mp\frac{A_0\left[B_2\exp\sqrt{\mp A_0} + B_3\exp(-\sqrt{\mp A_0})\right]^2}{B_1^2\mu^2\bar{f}^2\bar{S}^2} + \zeta^*_{cyl}\left(\frac{F_a}{F^*_a}\right)^2 \pm 1, \tag{7}$$

where

$$B_1 = 2A_0\left[\exp\sqrt{\mp A_0} - \exp(-\sqrt{\mp A_0})\right];$$

$$B_2 = 2A_0 + B_0\left[1 - \exp(-\sqrt{\mp A_0})\right];$$

$$B_3 = 2A_0 + B_0(1 - \exp\sqrt{\mp A_0});$$

$$B_4 = B_0\left[\exp\sqrt{\mp A_0} - \exp(-\sqrt{\mp A_0})\right].$$

Case II ($\mp A_0 < 0$):

$$\bar{Q}_x = \bar{W}_x = \frac{B_0}{2A_0}\left[\sqrt{1 + \frac{B_5^2}{\sin^2\sqrt{\pm A_0}}} \times \sin\left(\sqrt{\pm A_0}\,x + \operatorname{arctg}\frac{\sin\sqrt{\pm A_0}}{B_5}\right) - 1\right], \tag{8}$$

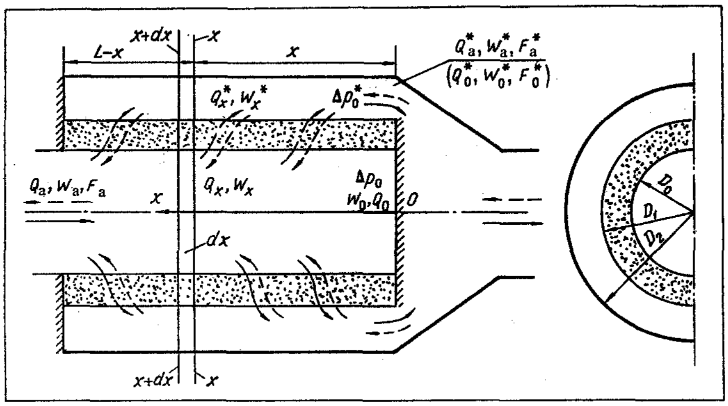

Fig. 1. Diagram of a cylindrical contact (or filtering) apparatus with Z-shaped flow.

$$\bar{v}_x = \frac{B_0 \sqrt{\pm A_0}}{2A_0} \sqrt{1 + \frac{B_5^2}{\sin^2 \sqrt{\pm A_0}}} \times$$

$$\times \cos\left(\sqrt{\pm A_0}\, x + \operatorname{arctg} \frac{\sin\sqrt{+A_0}}{B_5}\right), \qquad (9)$$

$$\Delta \bar{p}_x = \pm \frac{B_0^2 \left(1 + \frac{B_5^2}{\sin^2 \sqrt{\pm A_0}}\right)}{4A_0 \mu^2 \bar{f}^2 \bar{S}^2} \times$$

$$\times \cos^2\left(\sqrt{\pm A_0}\, x + \operatorname{arctg} \frac{\sin\sqrt{\pm A_0}}{B_5}\right), \qquad (10)$$

$$\zeta_{\text{tot}} = \pm \frac{B_0^2 \left(1 + \frac{B_5^2}{\sin^2 \sqrt{\pm A_0}}\right)}{4A_0 \mu^2 \bar{f}^2 \bar{S}^2} \times$$

$$\times \cos^2\left(\sqrt{\pm A_0} + \operatorname{arctg} \frac{\sin\sqrt{\pm A_0}}{B_5}\right) +$$

$$+ \zeta^*_{\text{cyl}}\left(\frac{F_a}{F^*_a}\right)^2 \pm 1, \qquad (11)$$

where

$$B_5 = \frac{2A_0}{B_0} + 1 - \cos\sqrt{\pm A_0}. \qquad (12)$$

Analysis of the above equations indicates that the expressions for $\bar{v}_{max}$ and $\bar{v}_{min}$, and therefore the extremal values of the abscissa $\bar{x}$, are different for different signs of the parameters A_0, B_0, B_1, B_2, B_3, B_4, and B_5.

Completely analogous formulas are obtainable in the case of tandem Z-shaped flow collectors (Fig. 2). This can be done simply by replacing the $\bar{f}\bar{S}$ (and $\bar{f}\bar{S}^*$) in the above formulas by

$$\bar{f}_{\text{br}} = \frac{\Sigma f_{\text{br}}}{F_a} \left(\text{and } \bar{f}^*_{\text{br}} = \frac{\Sigma f_{\text{br}}}{F^*_a}\right).$$

NOTATION

$\bar{Q}_x$, $\bar{W}_x$, and $\Delta\bar{p}_x$ are, respectively, the relative discharge rate (as a fraction of the initial discharge rate), the relative velocity (as a fraction of the initial velocity), and the relative static pressure (as a fraction of the initial dynamic pressure) in the inner cylinder, and also in the intake collector at a distance $\bar{x}$ from the origin; $\bar{v}_x$ is the relative velocity of flow through the porous surface or through a lateral branch of the collector (as a fraction of the average velocity over the entire layer surface or over all the lateral branches of the collector) at a distance $\bar{x}$ from the origin; ζ_{tot} is the total drag of the entire filtering (or contact) apparatus, and also of the entire tandem collector, as a fraction of the initial flow velocity; Fa

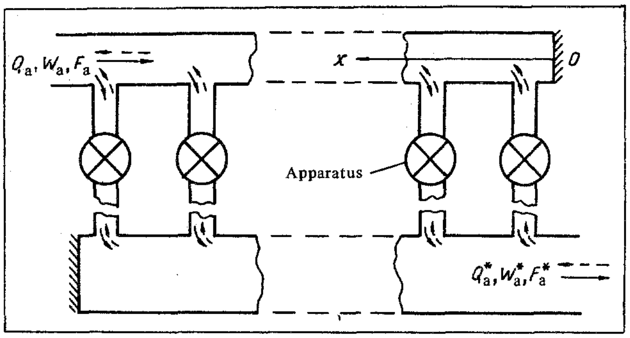

Fig. 2. Diagram of a tandem collector with Z-shaped flow.

and F^*_a are the cross-sectional areas of the intake and discharge channels of the apparatus (collector), in m^2; f_{br} is the cross-sectional area of a single branch of the collector, in m^2; $\bar{S}$ and $\bar{S}^*$ are the relative surface areas (as fractions of the initial areas) of one side of the intake and discharge segments of the apparatus or collector; f denotes the effective cross section (of the free volume) of the porous layer; ζ_{cyl} and ζ^*_{cyl} are the drag of the inner cylinder plus the intake segment of the collector, and the drag of the outer (annular) channel plus the discharge segment of the collector (each of these coefficients is given in first approximation by the formula $\zeta_{cyl} \cong 0.5\lambda\,(L/D_h)$ [where L/D_h is the relative length of the apparatus or collector; D_h is the hydraulic diameter of the segment; λ is the friction factor]); $\mu = 1/\zeta_{lay}^{1/2}$ is the coefficient of discharge through the layer or through a lateral branch of the collector as a fraction of the average velocity $\bar{v}_{av}$ in a lateral branch (or layer pores); ζ_{lay} is the drag of the porous layer or a lateral branch of the collector as a fraction of the velocity $\bar{v}_{av}$ (this drag is assumed in first approximation to be constant over the entire length of the apparatus (layer) or collector).

REFERENCES

1. I. E. Idel'chik, IFZh [Journal of Engineering Physics], 8, 5, 1965.

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Institute of Industrial and Sanitary Purification of Gases, Moscow